UDK 539.42
Original scientific article/Izvirni znanstveni članek

ISSN 1580-2949
MTAEC9, 40(4)157(2006)

ESTIMATION OF THE FATIGUE THRESHOLD VALUES FOR A CRACK PROPAGATING THROUGH A BI-MATERIAL INTERFACE TAKING INTO ACCOUNT RESIDUAL STRESSES

OCENA UTRUJENOSTNEGA PRAGA ZA RAZPOKO, KI NAPREDUJE SKOZI VMESNO PLOSKEV MED DVEMA MATERIALOMA Z UPOŠTEVANJEM REZIDUALNIH NAPETOSTI

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Prejem rokopisa – received: 2006-05-17; sprejem za objavo – accepted for publication: 2006-07-18

This article deals with the behaviour of a fatigue crack propagating across a bi-material interface. A stability criterion for the crack touching the bi-material interface, taking into account the residual stresses closing the crack faces, is formulated. Linear elastic fracture mechanics are assumed to apply and the finite-element method is used in the calculations. The criterion proposed is applied to determine the fatigue threshold stress for crack propagation across the interface. Two different geometries are considered. It is shown that the threshold values for crack propagation are influenced by the residual stresses closing the crack and by the specific combination of the elastic constants of the materials used. The results contribute to a better understanding of the failure of structures with bi-material interfaces (protective layers, composite materials, etc.).

Key words: bi-material interface, residual stress, plasticity-induced crack closure, critical stress

Članek obravnava utrujenostno razpoko, ki napreduje skozi ploskev med dvema materialoma. Formulirano je merilo za razpoko, ki se dotakne vmesne ploskve, ki upošteva rezidualno napetost, ki zapira ustnici razpoke. Za izračune sta uporabljeni linearna mehanika loma in metoda končnih elementov. Merilo je uporabljeno za določitev praga napetosti za prehod razpoke preko vmesne ploskve. Upoštevani sta dve geometriji. Znano je, da na napetost na pragu vplivajo rezidualne napetosti, ki zapirajo razpoko, in specifična kombinacija elastičnih konstant obeh materialov. Rezultati omogočajo boljše razumevanje zloma struktur z vmesnimi ploskvami (varovalni sloji, kompozitni materiali itd).

Ključne besede: vmesna ploskev, rezidualne napetosti, plastično zaprtje razpoke, kritična napetost

1 INTRODUCTION

The interface between two dissimilar media represents a weak point for many applications of structures composed of different materials. The presence of regions with different mechanical properties and the existence of an interface between them have a pronounced influence on the stress distribution of composite bodies. The characteristics of fracture in the vicinity of, and through, the interface are influenced strongly by the properties of the interface and of the materials on either side of the interface. Fracture usually starts at a defect in the interface, especially at an interface microcrack, or at the edge of the interface. Another important factor is the influence of the interface on a crack penetrating that interface from one material into the second in a bi-material body.

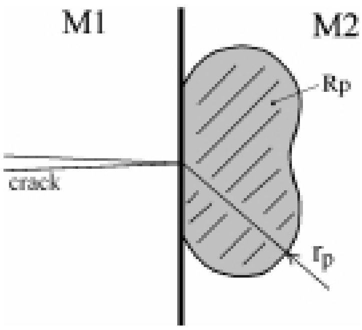

Figure 1: The plastic zone created by a crack with its tip at the interface. Material properties are described by the Young's modulus *E*, the Poisson's ratio ν and the yield stress σ_0 of each material.

Slika 1: Plastična zona zaradi razpoke z vrhom na vmesni površini. Lastnosti materiala opisujejo Youngov modul *E*, Poissonovo število ν in meja plastičnosti σ_0 za vsak material

2 A CRACK TERMINATING PERPENDICULAR TO THE INTERFACE

The stress distribution around the crack tip (in the case of a crack perpendicular to the interface) can be expressed in its general form, as follows (e.g., [1,2]):

$$\sigma_{ij} = \frac{H_1}{\sqrt{2\pi r^{p}}} f_{ij}(p,\alpha,\beta) \qquad (1)$$

where $f_{ij}(p,\alpha,\beta)$ is a known function of the bi-material parameters α and β, as defined in [1], and $0 < p = p(\alpha,\beta) < 1$ is the stress singularity exponent. For given materials and loading conditions the stress distribution around the crack tip is determined by the value of the generalised stress-intensity factor H_I. The value of H_I is proportional to the applied load and has to be estimated numerically. For homogeneous materials, $H_I = K_I$ is the stress-intensity factor, and $p = ½$.

The stress distribution around a crack with its tip at the interface of two different elastic materials as given by Eq. (1) represents a general singular stress concentrator. The fact that the stress singularity exponent differs from ½ means that the linear elastic fracture mechanics procedures and criteria cannot be used [3].

In a homogeneous body (we consider for simplicity that the corresponding value of the load ratio $R = 0$, i.e., $\Delta K_I = K_I$) and under the conditions corresponding to a high cycle fatigue, the fatigue threshold condition has the form

$$K_I(\sigma_{\mathrm{th}}^{hom}) = K_{\mathrm{th}} \tag{2}$$

The fatigue crack will not propagate if the value of the applied load, expressed in terms of the stress intensity factor range K_I, is less than the corresponding threshold value of the material K_th.

Similarly, for a crack with its tip at the interface, the stability condition can be written in the form

$$H_\mathrm{I}(\sigma_{\mathrm{th}}) = H_{\mathrm{th}}(K_{\mathrm{th}}) \tag{3}$$

In other words, the fatigue crack stays arrested at the interface if the value of the applied load, expressed in terms of the generalised stress-intensity factor range H_I, is less than the corresponding generalised fatigue threshold value of the material H_th. The generalised threshold value H_th is a function of the fatigue threshold value K_th of the material M2 and, moreover, it depends on the elastic mismatch of the materials M1 and M2, as expressed by the bi-material parameters α,β, i.e., $H_\mathrm{th} = H_\mathrm{th}(K_\mathrm{th},\alpha,\beta)$, see [4].

Instead of the values K_th and H_th the fatigue threshold stress can be used as the quantity describing the behaviour of the crack. The threshold stress σ_th is the value of the externally applied tensile stress σ_appl at which the crack will start to grow. The fatigue threshold condition for a propagating crack is:

$$\sigma_\mathrm{appl} < \sigma_\mathrm{th} \tag{4}$$

A method introduced in previous publications (see [4] for details) was used to determine the generalised threshold values H_th and the threshold stress σ_th. The method, which is based on an assessment of the plastic zone size (see **Figure 1**), yields the following expression for the critical value of the generalised stress-intensity factor:

$$H_{\mathrm{th}}(K_{\mathrm{th}}) = K_{\mathrm{th}}^{2\,\mathrm{p}}\sigma_0^{(1-2\,\mathrm{p})}\left[\frac{f_{\mathrm{hom}}(\nu)}{f(\alpha,\beta,\nu)}\right]^{\frac{\mathrm{p}}{2}} \tag{5}$$

where p is the stress singularity exponent, K_th is the threshold value of the stress-intensity factor determined for the material M2 into which the crack is to grow, σ_0 is the yield stress of the material M2, and the expression within the square brackets represents the ratio of the areas of the plastic zones in front of the crack tip in a homogeneous material (only the material M2 is considered, see **Figure 1**), and in the real bi-material case. $\nu = \nu_2$ is the Poisson's ratio of the material M2. The threshold stress is then given by:

$$\sigma_{\mathrm{th}} = \frac{H_{\mathrm{th}}\sigma_{\mathrm{appl}}}{H_I(\sigma_{\mathrm{appl}})} \tag{6}$$

where H_th is the threshold value determined from equation (5) and $H_\mathrm{I}(\sigma_\mathrm{appl})$ is the generalised stress-intensity factor determined numerically (using the FEM) for an applied tensile stress σ_appl.

The procedure mentioned above and the results obtained from the criterion (6) neglect the existence of a reversed plastic zone and the closure of the fatigue crack during propagation. The phenomenon of fatigue-crack closure was investigated and described by Elber [5] as so-called "plasticity induced crack closure". Numerical elasto-plastic FEM calculations can be used to estimate the level of the applied external loading in the sense of Newman's calculations of opening stresses, e.g., [6]. For simplicity, a pulsating (sinusoidal) loading is assumed. The effective value of the threshold stress is then:

$$\sigma_{\mathrm{th}}^{\mathrm{eff}} = \sigma_{\mathrm{th}} + \sigma_{\mathrm{op}} \tag{7}$$

where $\sigma_{\mathrm{th}}^{\mathrm{eff}}$ is the effective threshold stress taking into account the plasticity-induced crack closure, σ_th is the threshold stress given by Eq. (5), and σ_op is the

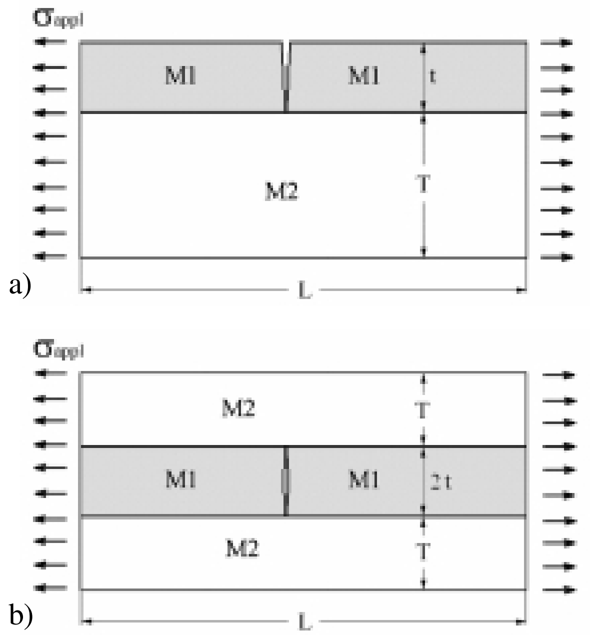

Figure 2: The bi-material body with "edge" (a) and "central" (b) crack under tensile loading considered in the numerical example. T = 25 mm, t = 12.5 mm, L = 75 mm.

Slika 2: Telo iz dvojnega materiala z robno (a) in centralno (b) razpoko pri natezni obremenitvi, upoštevani pri numeričnem primeru: T = 25 mm, t = 12.5 mm, L = 75 mm

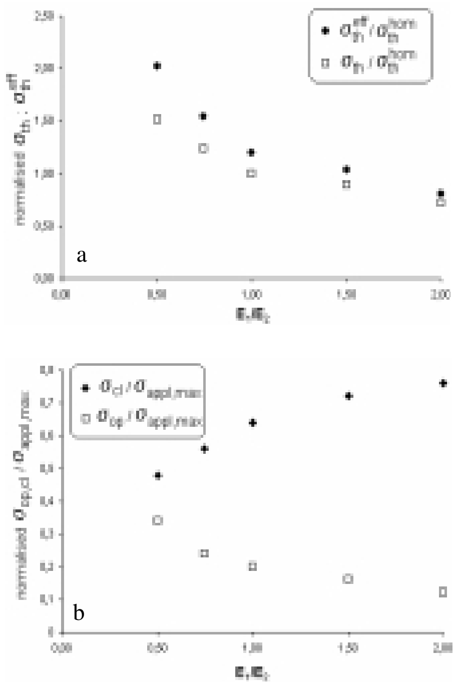

Figure 3: a) Dependence of the normalised effective threshold stress on Young's modulus ratio; b) normalised ($\sigma_{op,cl}$ /σ_{appl}) values of the opening/closing stresses as a function of the Young's modulus ratio for a bi-material body with an "edge" crack.

Slika 3: a) Odvisnost normaliziranega dejanskega praga napetosti od razmerja Young modulov; b) normalizirane ($\sigma_{op,cl}$/σ_{appl}) vrednosti za napetosti odprtja in zaprtja v odvisnosti od Young modula za telo iz dveh materialov z robno razpoko

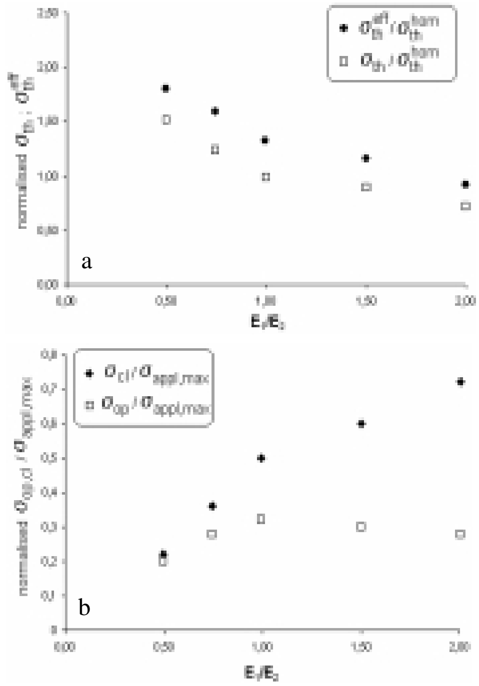

Figure 4: a) Dependence of the normalised effective threshold stress on Young's modulus ratio; b) normalised ($\sigma_{op,cl}$ /σ_{appl}) values of the opening/closing stresses as a function of the Young's modulus ratio for a bi-material body with a "central" crack.

Slika 4: a) Odvisnost normaliziranega dejanskega praga napetosti od razmerja Young modulov; b) normalizirane ($\sigma_{op,cl}$/σ_{appl}) vrednosti za napetosti odprtja/zaprtja v odvisnosti od razmerja Young modulov za telo iz dveh materialov s centralno razpoko

computed opening stress [7]. Eq. (7) can also take into account various loading ratios, R.

3 NUMERICAL EXAMPLE

The proposed procedure is applied to the estimation of the threshold stress σ_{th} corresponding to the threshold level in the case of cracked bi-material bodies (see **Figure 2**). First, the procedure published in [4] and Eq. (6) were used to determine σ_{th}. Then the model was subject to cyclic loading with a loading ratio of $R = 0$. The crack tip was located 0.3 mm in front of the interface at the beginning of the calculation and directly at the interface at the end of the calculation. Six load cycles were modelled, with the crack length increasing by 0.05 mm at each step. From the numerical calculation the opening (and closing) stress σ_{op} (and σ_{cl}), were determined, see **Figure 3a and 4a**, and from Eq.7 the value of σ_{th}^{eff} was calculated. The results for different values of E_1/E_2 and two kinds of geometries are shown in **Figure 3 a, b and Figure 4 a, b**.

The results correspond to the plane strain approximation, K_{th} = 5 MPa.m$^{1/2}$, σ_0=280 MPa. The values σ_{th} and σ_{th}^{eff} were normalised with the value σ_{th}^{hom}, corresponding to the fatigue threshold stress obtained from Eq. 2. σ_{th}^{eff} corresponds to the threshold stress taking into account the plasticity-induced crack closure in terms of Eq. 7. The Chaboche material model (kinematic hardening) was used for both materials with the same hardening parameters. The material M2 corresponds to a steel with σ_0 = 280 MPa. A loading ratio of R = 0 and an amplitude of 80 MPa of pulsating tensile stress were applied.

4 SUMMARY

The influence of a bi-material interface between two different materials on the fatigue threshold of a crack has been investigated. Numerical elasto-plastic calculations were performed for a crack perpendicular to the bi-material interface, and the opening stress was determined.

A previously proposed tentative procedure, modelling the propagation of a fatigue crack through the interface between two materials, was used in the paper. Attention was focused on the case of a fatigue crack that touches the interface. The above procedure makes it possible to quantify the effect of the interface on the fatigue threshold value for crack propagation from the one material into the other. The procedure is based on an extension of linear elastic fracture mechanics to general singular stress concentrators.

The threshold values obtained from the elastic calculations were supplemented and corrected by elastic-plastic calculations of the opening stress, and the effective threshold stress for crack propagation across the bi-material interface was obtained. Two different geometries were investigated.

It follows from the results presented here that the corresponding fatigue threshold value can be strongly influenced by the presence of a material interface. The correction for the existence of a plastic wake behind the tip of a propagating fatigue crack allows for an estimation of the threshold values for crack propagation through the bi-material interface. The results obtained show (see **Figure 3a and 4a**) that for softer material M1 than M2 ($E_1<E_2$) the fatigue threshold value increases. In opposite case ($E_1>E_2$), the fatigue threshold value decreases in comparison to the homogeneous material, with elastic properties of material M2. The results for the case of "edge" and "central" crack are more or less similar, but it is clear that the influence of Young's modulus ratio is stronger for case of the "edge" crack.

Figures 3b and 4b show the ratio between the maximum of the applied stress and the opening (closing) stresses. The differences between the individual cases are caused by different geometries and boundary conditions.

The results obtained contribute to a better understanding of the damage that such cracks may cause in composite materials.

Acknowledgements

This research was supported by the grant No. 106/04/P084 of the Czech Science Foundation.

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