

# RELATIONSHIP BETWEEN MECHANICAL STRENGTH AND YOUNG'S MODULUS IN TRADITIONAL CERAMICS

## ODVISNOST MED MEHANSKO TRDNOSTJO IN YOUNGOVIM MODULOM PRI TRADICIONALNI KERAMIKI

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*Prejem rokopisa – received: 2010-10-04; sprejem za objavo – accepted for publication: 2011-05-27*

A verification of theoretical linearity between mechanical strength and Young's modulus was performed with quartz porcelain samples both green and fired. The experiments were carried out at the room temperature and at elevated temperatures up to 1000 °C. The results obtained for green samples showed relatively scattered values  $\sigma_f(E)$  around the linear function. A regression coefficient of the linear fitting ( $R = 0.803$ ) is not sufficient for a clear linearity  $\sigma_f(E)$ . The relationship  $\sigma_f(E)$  is clearly linear for a fired sample and was confirmed in the temperature interval 20–1000 °C. For this case, the regression coefficient of the linear fitting is 0.976.

Key words: ceramics, Young's modulus, flexural strength, firing

Teoretična linearnost med mehansko trdnostjo in Youngovim modulom je bila preverjena na vzorcih iz kremenovega porcelana, zelenih in žganih. Poizkusi so bili izvršeni pri sobni temperaturi in pri povišanih temperaturah do 1000 °C. Pri rezultatih iz zelenih vzorcev je imela vrednost  $\sigma_f(E)$  precejšnje odmike od linearnosti. Regresijski koeficient linearnega ujemanja ( $R = 0.803$ ), kot jasno merilo linearosti  $\sigma_f(E)$ , je premajhen. Odvisnost  $\sigma_f(E)$  je jasno linearna za žgane vzorce in je bila potrjena za vzorce, žgane v intervalu temperature 20–1000 °C. Za žgane vzorce je koeficient regresije 0.976.

Ključne besede: keramika, Youngov modul, upogibna trdnost, žganje

## 1 INTRODUCTION

Mechanical parameters are important characteristics of ceramic materials. Each ceramic product is mechanically stressed in technological processes during drying and firing and, as well as in actual service. Both flexural strength and Young's modulus are among the most important physical parameters of ceramic material and appear in theoretical models and calculations related to permissible loading the ceramic products. They also play crucial roles (together with a coefficient of thermal expansion and coefficient of thermal conductivity) in the calculation of the maximum firing rate.

The linear relationship between mechanical stress and strain follows directly from Hooke's law <sup>1</sup>. In the simplest one-dimensional case, the Hooke's law takes a form of  $\sigma = E\varepsilon$ , where  $\sigma = F/S$  is the stress ( $F$  is a loading force and  $S$  is area of the sample cross-section) and  $\sigma = \Delta l/l$  ( $\Delta l$  is extension/contraction of the sample and  $l$  is its initial length). A quantity  $E$  is Young's modulus which characterizes elastic properties of the sample material.

A measurement of Young's modulus based directly on the equation  $\sigma = E\varepsilon$  requires a relatively high stress to reach a measurable deformation. It can influence the structure of the tested material and create microcracks in

brittle materials such as ceramics. For that reason, a flexion of the sample is often used for ceramic materials. For example, by static three-point-bending, Young's modulus  $E$  and mechanical strength  $\sigma_f$  are determined by relations <sup>2</sup>

$$E = \frac{4Fl^3}{3\pi d^4 y}, \sigma_f = \frac{8F_f l}{\pi d^3} \quad (1a, 1b)$$

for a circular cross-section, where  $y$  is a flexion in the middle between the supports,  $l$  is a support span,  $d$  is a diameter of the sample and  $F_f$  is the loading force, at which a rupture occurred. These equations combined, provides the following

$$\sigma_f = \frac{6d}{l^2} y_f E \quad (2)$$

where  $y_f$  is flexion of the sample at the instant of the rupture. In analyzing other methods of simultaneous measuring of the mechanical strength and Young's modulus, the similar result is obtained. There is linearity between the mechanical strength and Young's modulus

$$\sigma_f = Ky_f E \quad (3)$$

where constant  $K$  contains dimensions of the sample and specific geometrical parameters of mechanical design of an experiment. The character of the parameter  $y_f$  is given via experiment, e.g. it is flexion (as in the described

example), or extension (contraction) of the sample in different experiments.

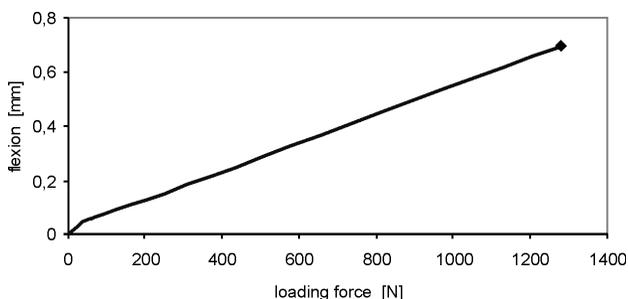
In contrast to the modulus of elasticity, values of flexural strength depend on the method used and on the dimensions of the sample. Ceramic material is characterized by fragility. Ceramic samples under mechanical load exhibit Hooke's law until reaching the critical deformation when a rupture of the sample occurs. A typical relationship between the flexion and loading force of the porcelain sample in the three-point-bending test is depicted in **Figure 1**.

Hypothetically, linearity between the mechanical strength and Young's modulus is valid when Young's modulus is measured, not simultaneously with mechanical strength, and even when different methods are used for their measurement, e.g., the static method for mechanical strength and dynamical method for Young's modulus.

The mechanical strength depends on the crack initiator presence in the most loaded area (e.g., in the middle of the sample if the three-point-bending is used). These initiators are not identical and produce the rupture of the samples at different loading forces and brings relatively high scatter of values of mechanical strength<sup>2,3</sup>. A relationship between the mechanical strength and the size of the sample is also known<sup>2,3,4,5</sup>. This property of the mechanical strength requires a relatively high number of samples. If a temperature dependence of the mechanical strength is required, e.g. for 10 temperatures, more than one hundred of samples must be used.

On the other side, elastic modulus is an integral value which does not depend on accidental occurrence of the big crack in the some peculiar place of the sample. That is, if some number of the samples is measured, the elastic modulus varies only in a small extent. Thus, we need substantially less number of samples for the measuring the elastic modulus than the mechanical strength. Beside that, the elastic modulus does not depend on the sample size.

An advantage of the measurement of the elastic modulus comparing to the mechanical strength can lead to a suggestion to utilize the linear relationship between these qualities and substitute the measurement of the



**Figure 1:** Dependence of the flexion on the loading force. Point (◆) corresponds with a rupture

**Slika 1:** Odvisnost med upogibom in obremenitvijo. Točka (◆) je označba za prelom

mechanical strength with the measurement of Young's modulus and having the value of Young's modulus, calculate the mechanical strength according to equation  $\sigma_f = \text{const} \cdot E$ . But we have never met such procedure. The linear relationship  $\sigma_f(E)$  was used for rejection of ceramic components with substandard mechanical properties<sup>6</sup>.

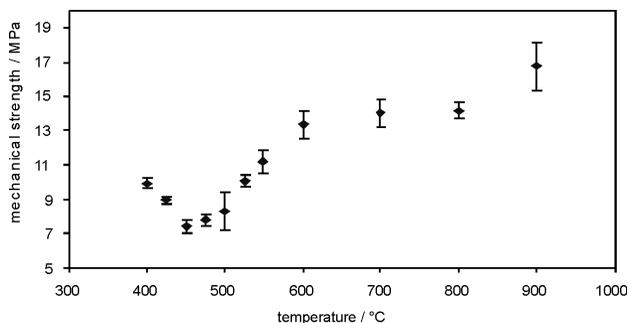
In our previous work<sup>7</sup>, we found the constant of proportionality in equation  $\sigma_f = \text{const} \cdot E$  for porcelains with reference to data given by porcelain manufacturers and research laboratories. The regression function for this relationship is  $\sigma_f \approx 1.21 \cdot 10^{-3} E$ ,  $R^2 = 0.6354$ , where  $\sigma_f$  is in MPa and  $E$  is in GPa. By using this relationship it is possible to evaluate approximately the flexural strength or Young's modulus if one of them is known. However, poor regression coefficient, which is a consequence of the different values taken from different sources, does not allow use equation  $\sigma_f = \text{const} \cdot E$  for sufficiently faithful and accurate conversion of the Young's modulus into the mechanical strength.

The objective of this paper is verification of equation  $\sigma_f = \text{const} \cdot E$  for green porcelain mixture during its firing and after the firing.

## 2 EXPERIMENTAL

Samples were made from a plastic mass of the mass fractions 50 % kaolin and clay, 25 % quartz, 25 % feldspar and water for manufacturing quartz porcelain high-voltage insulators. The cylindrical samples were made with the laboratory extruder. After drying in the open air, the samples contained  $\approx 1$  % of physically bounded water. The final dimensions of the green sample for thermomechanical analysis (mf-TMA) after drying, were  $\Phi 11$  mm  $\times$  150 mm and  $\Phi 11$  mm  $\times$  120 mm for flexural strength test. The volume mass of the green sample material 1822 kg/m<sup>3</sup> was determined from the sample weight and dimensions.

Young's modulus was measured by a non-destructive sonic resonant technique – sensitive and reliable at elevated temperatures<sup>8</sup>. This method is based on measuring the resonance frequency, which is used for the calculating of Young's modulus, if the volume mass and



**Figure 2:** Mechanical flexural strength measured at the actual temperature

**Slika 2:** Upogibna trdnost, izmerjena pri naraščajoči temperaturi

dimensions of the sample are known. Using a flexural vibration, Young's modulus may be calculated for a cylindrical sample with a uniform square cross-section with the formula <sup>1,9</sup>

$$E = 1.26193 \left( \frac{l^2 f}{d} \right)^2 \rho T \quad (4)$$

where  $f$  is a resonant frequency of the fundamental mode,  $\rho$  is a volume mass,  $l$  is the length and  $d$  is the diameter of the sample. A value  $T$  is a correction coefficient, to be used if  $l/d < 20$ . For  $l/d = 15$  and Poisson's ratio  $\mu \approx 0.2$ , the correction coefficient  $T$  was taken from a table given in <sup>1</sup>,  $T = 1.01983$ .

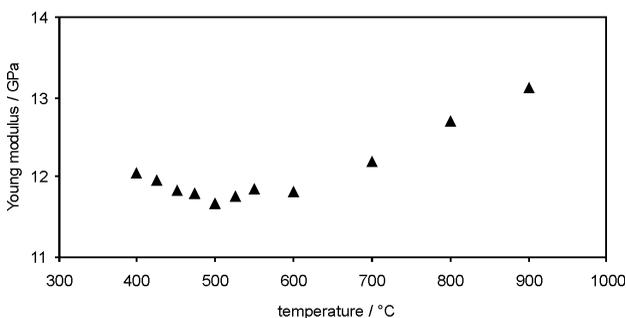
Mechanical strength  $\sigma_f$  was determined by the three-point-bending test from Eq. (1b) at elevated temperatures during heating as well as at room temperature.

### 3 RESULTS AND DISCUSSION

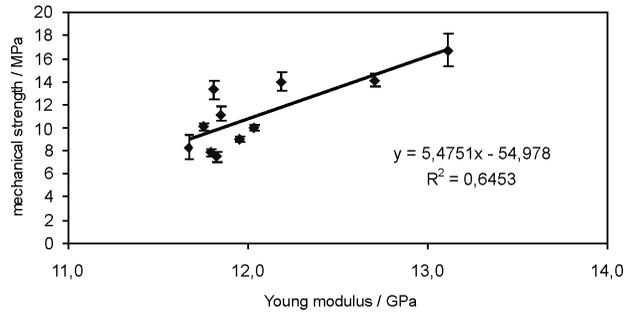
Two experiments were performed. In the first, green samples were heated with a rate 5 °C/min and broken at the temperatures (400, 425, 450, 475, 500, 550, 600, 700, 800 and 900) °C in a regime of a constant rate of the loading force, 2 N/s. The results shown in **Figure 2** are similar to the results presented in <sup>10</sup>.

The green sample was also subjected to modulated-force mechanical thermal analysis (mf-TMA) to obtain values of Young's modulus at the temperatures referred to above. The Young's modulus was calculated from Eq. (4), where the resonant frequency was measured. The dimensions and mass of the sample assumed to be constant. The relationship Young's modulus versus temperature is depicted in **Figure 3**.

The relationship between mechanical strength and Young's modulus was verified, see **Figure 4**. The courses of graphs in **Figure 2** and **Figure 3** are similar, but the expected linear function, see **Figure 5**, is only approximately valid. In addition, the regression function in **Figure 4** does not fulfill a requirement  $\sigma_f \rightarrow 0$ , if  $E \rightarrow 0$ . A cause of the relatively low value of the regression coefficient of the linear fitting,  $R = 0.803$ , is uncertain up to now.

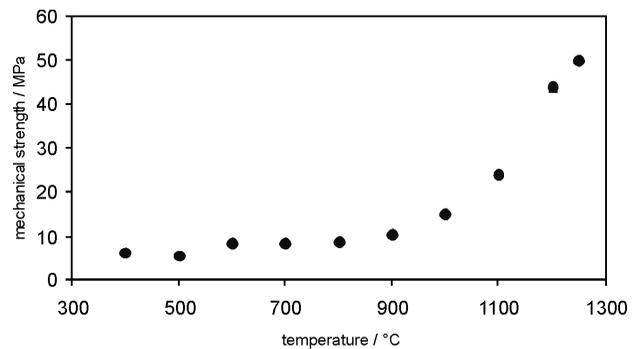


**Figure 3:** Young modulus measured at the actual temperature  
**Slika 3:** Youngov modul, izmerjen pri naraščajoči temperaturi



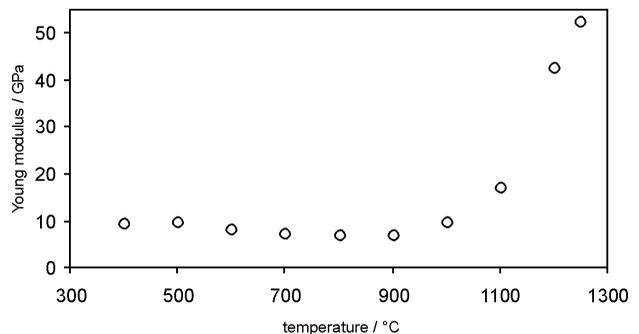
**Figure 4:** Relationship between mechanical strength and Young modulus measured at the actual temperature

**Slika 4:** Odvisnost med mehansko trdnostjo in Youngovim modulom pri različni temperaturi



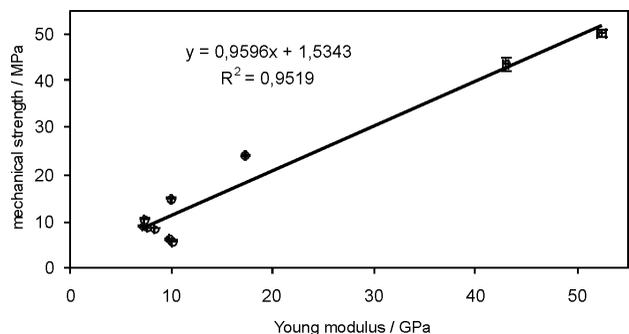
**Figure 5:** Dependence of the mechanical flexural strength on the firing temperature

**Slika 5:** Odvisnost upogibne trdnosti od temperature žganja



**Figure 6:** Dependence of Young modulus on the firing temperature

**Slika 6:** Odvisnost med Youngovim modulom in temperaturo žganja



**Figure 7:** Relationship between mechanical strength and Young modulus measured at the room temperature

**Slika 7:** Odvisnost med mehansko trdnostjo in Youngovim modulom, izmerjena pri sobni temperaturi

In the second experiment, sets of 8 green samples were heated up to (400, 500, 600, 700, 800, 900, 1000, 1100, 1200 and 1250) °C with a rate 5 °C/min and then freely cooled in the oven. The sets of 8 samples were used for measuring Young's modulus and then for measuring the mechanical strength, both at room temperature. The results are displayed in **Figure 5, 6**, where the relationship between the mechanical strength and Young's modulus versus firing temperature is shown. A high similarity can be observed between these graphs. A relationship presented in **Figure 7** is very close to the linear dependence. The regression coefficient of the linear fitting  $R = 0.976$  is high and confirms the linearity between these material properties and the physically correct condition of  $\sigma_f \rightarrow 0$ , if  $E \rightarrow 0$  is nearly met. This permits using the dynamical measurement of Young's modulus of one sample rather than the measurement of the mechanical strength which typically requires more than 15 samples.

#### 4 CONCLUSION

A verification of theoretical linearity between mechanical strength and Young's modulus was performed with quartz porcelain samples, both green and fired. The experiments were carried out at room temperature and at elevated temperatures up to 1000 °C.

The results obtained for green samples showed relatively scattered values  $\sigma_f(E)$  around the linear function. Thus a regression coefficient of the linear fitting ( $R = 0.803$ ) is not sufficient for a conclusion with regard of a strong linearity  $\sigma_f(E)$ .

The relationship  $\sigma_f(E)$  is clearly linear for fired sample and confirmed in the temperature interval

20–1000 °C by the regression coefficient of the linear fitting of 0.976.

**Acknowledgements:** This work was supported by the grants VEGA 1/0216/09 and APVV SK-CZ-0005-09 and the Ministry of Education, Youth and Sports of the Czech Republic, under the project No. MSM: 6840770031. The authors thank the ceramic plant PPC Čab for providing green ceramic samples.

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