

ONE-DIMENSIONAL MATHEMATICAL MODELING OF THE CAPSULE FLOW IN A HORIZONTAL PIPE

ENODIMENZIONALNO MATEMATIČNO MODELIRANJE PRETOKA LEDENIH KAPSUL V HORIZONTALNEM CEVOVODU

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Horizontal hydraulic transport of ice particulates suspended in water is a well-known practice used for cooling systems in industry. Many advantages of hydraulic transport include its environmentally friendly application and a relatively low cost of maintenance and operation. This study presents a one-dimensional mathematical model, developed for simulating a pressure drop of solid and water two-phase fluid flow inside a horizontal pipe. The impact of different parameters such as suspension-wall friction coefficients, suspension-viscosity models and solid volumetric concentrations on the pressure drop of the capsule flow were evaluated. The results of the simulations were validated by the results of two different experiments. The first experiment was performed with anomalous-shaped capsules, which had a higher density than the density of the liquid flow while the second experiment was performed with capsules having a spherical shape and a density lower than the density of the carrier liquid flow in the pipe. The measurements were carried out in the horizontal pipe. In both experiments, the carrier fluid was water. The results of this modeling study revealed that the proposed model can be effectively used to predict the pressure drop of ice and water two-phase flow with sufficient precision.

Keywords: two-phase flow, pressure drop, capsule flow, suspension viscosity

Horizontalni hidravlični transport delcev ledu v vodni raztopini je dobro znana praksa, uporabljena v industrijskih hladilnih sistemih. Hidravlični transport je okolju prijazen in ima mnoge prednosti zaradi relativno nizkih stroškov obratovanja in vzdrževanja. Avtorji predstavljajo študijo enodimenzionalnega matematičnega modela, ki so ga razvili za simulacijo padca tlaka v horizontalnem cevovodu med pretakanjem dvofaznega medija (suspenzija trdno/kapljevina). Ovređnotili so vpliv različnih parametrov, kot so koeficienti trenja med steno in suspenzijo, modelov za viskoznost suspenzije in volumske koncentracije na padec tlaka med pretokom ledu v obliki kapsule. Veljavnost rezultatov simulacij so ocenili s pomočjo podatkov, dobljenih z dvema različnima eksperimentoma. Prvi eksperiment so izvedli z nepravilno obliko kapsul, ki ima višjo gostoto kot pretok same kapljevine. Drugi eksperiment pa je bil izveden s kroglično obliko kapsul in nižjo gostoto, kot jo ima dejanski nosilni medij v cevovodu. Vse eksperimentalne meritve so bile izvedene v horizontalnem cevovodu. V obeh primerih je bila voda nosilni medij. Rezultati modelnih študij so pokazali, da lahko predlagani model z zadovoljivo natančnostjo učinkovito uporabimo za napoved padca tlaka pri pretoku dvofaznega (led in voda) medija.

Ključne besede: dvofazni pretok, padec tlaka, pretok kapsul, viskoznost suspenzije

1 INTRODUCTION

Hydraulic transport of capsules in pipelines have become a subject of intense research in recent decades. Capsule transportation through horizontal pipelines is employed in many engineering processes.¹⁻⁹ Two-phase flow of ice and water in cylindrical tubes is most widely used in cooling systems in industry.¹⁰ The use of ice-water slurries in district cooling systems can be considered as one of the most crucial technical improvements for the industrial development. In addition, global-warming impacts of chemical refrigerants, especially ozone-depletion concerns caused the refrigeration industry to search for environmentally friendly refrigerant options.¹¹ Moreover, ice-water slurry has a much higher energy capacity compared to the conventional chilled-water systems and it produces a much greater cooling potential at the same pumping cost.¹² On the

other hand, using ice capsules is much more advantageous compared to ice slurries due to a lack of dispersion of the particles leading to a blockage in the pipes. The cooling capacity in district systems can be improved by using ice capsules instead of ice slurries due to their high concentration and flow velocity. In the literature, several studies focused on the hydrodynamic behavior of a two-phase ice-water flow^{1,4-10,13-16} and also on the transfer of heat during the capsule flow.^{12,17-20}

Vast studies on two-phase ice-water flow experiments are available in the literature; however, investigations on the characterization of flow behavior are very limited. This is mainly due to high complexity of the mathematical modeling of two-phase flow such as ice and water.^{7,17,21,22} Intensive research is required to precisely understand and be able to make prediction on the flow behavior, and to search for the ways of improving their processing. Therefore, the motivation behind this study was developing a one-dimensional mathematical model to simulate the pressure drop of the flow of an ice-water

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two-phase mixture inside a horizontal pipe. In the modeling study, the parameters having a high impact on the pressure drop of a capsule flow system such as suspension-viscosity model, suspension-wall friction coefficient and volumetric concentrations were investigated. The results of the developed mathematical model were validated with two different sets of experimental data obtained from the literature. The first set of data¹³ (A) referred to a capsule flow of an anomalous shape having a higher density than the fluid, and the second set of data¹⁵ (B) was adopted from an experiment performed with spherical-shaped capsules having a lower fluid density.

2 MODEL DESCRIPTION

Modeling of two-phase flows such as ice and water is a challenging process due to the fact that it depends on many factors, resulting in a wide range of flow regimes. Therefore, developing a simplified model, which can describe the two-phase flow of an ice-water system with a suitable precision is necessary. In this respect, a one-dimensional mathematical model is developed and solved with the Gauss-Seidel method using MATLAB.

The assumptions presented below were made to solve the governing equations in the model for the capsule-carrying pipeline:

Assumption 1: Homogenous flow

Assumption 2: Fully developed flow

Assumption 3: The influence of the heat transfer/pressure drop along the pipe on the solid-phase concentration is negligible

Assumption 4: Buoyancy effects of the phases differ

Assumption 5: The suspension-wall friction is independent of the water temperature

Figure 1 shows a schematic illustration of an ice-water flow. As can be seen from the figure, a control volume is defined within the pipe where the ice-water two-phase flow is modeled.

A flow chart of the model is presented in **Figure 2**. The figure includes all the governing equations and cor-

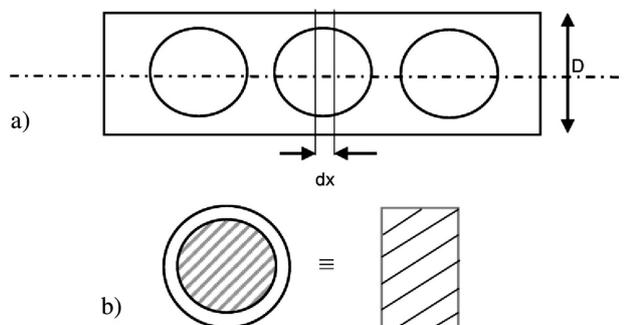


Figure 1: Schematic illustration of the spherical capsule in a pipe flow: a) the side view of the pipe, b) the front view of the pipe and its equivalent control volume

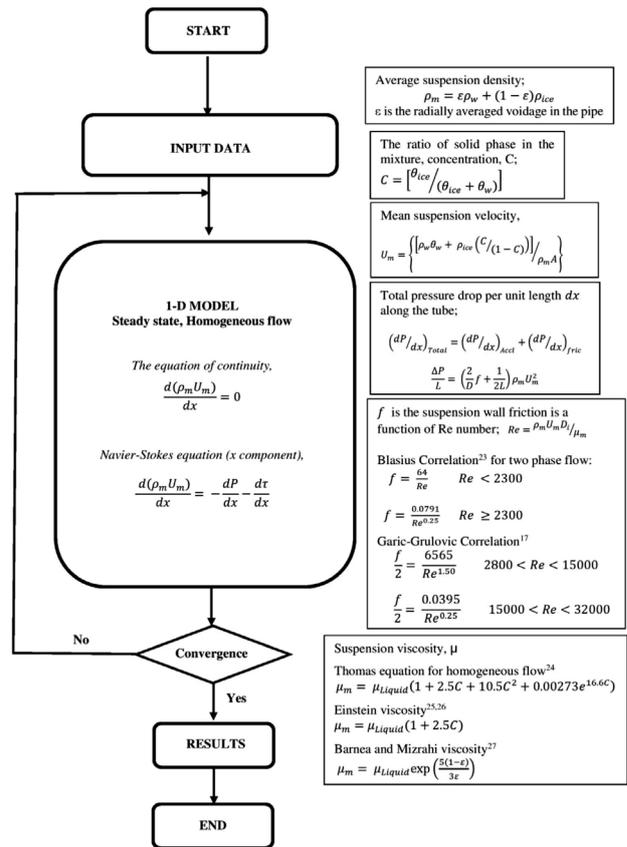


Figure 2: Flow chart of the modeling study

relations for the steady-state one-dimensional homogeneous equilibrium flow in a horizontal pipe.

3 RESULTS AND DISCUSSION

The mathematical model of a homogenous two-phase flow developed in this study was validated with the published experimental data.^{13,15} In the experiments, two-phase capsule-flow measurements were carried out in a horizontal straight pipe (plexiglass) where water was the carrier fluid. In the first set of experiments,¹³ the accuracy of the liquid velocity was found to be about 2.5 %, whereas the accuracy of the capsule velocity was 1.5 %. During the pressure-drop measurements, the accuracy of was about 3 %. Anomalous-shaped capsules having their density greater than that of the carrier liquid were used in these experiments. With the second data set,¹⁵ spherical capsules, fabricated from polypropylene having their density lower than that of the water were used. In this study, the flow rate was measured with water flow meters having an average deviation of 0.5 % (max. 0.1 %) from the actual values. The measurements of the pressure drop were carried out on a 4-m section of the pipe. The ratio of the capsule diameter to the pipe diameter (d/D) was 0.8. The main parameters used in the experiments and the conditions are given in **Table 1**.

Table 1: Parameters used in experimental studies

	Experiment A ¹³	Experiment B ¹⁵
Capsule geometry	Anomalous	Spherical
L (m)	2	4
D (m)	0.05	0.1
d/D (-)	0.8	0.8
ρ_c (kg/m ³)	1200	870
C (-)	0.20-0.50	0.05-0.30
Reynolds number (-)	$2.5 \times 10^4 < Re < 1.5 \times 10^5$	$1.2 \times 10^4 < Re < 1.5 \times 10^5$
Fluid temperature (°C)	20	20

For the model validations, the suspension viscosity and suspension-to-wall friction values were calculated with the formulas given in **Figure 2**.

The comparison of the simulation results for the first experimental data are shown in **Figure 3**. As can be seen

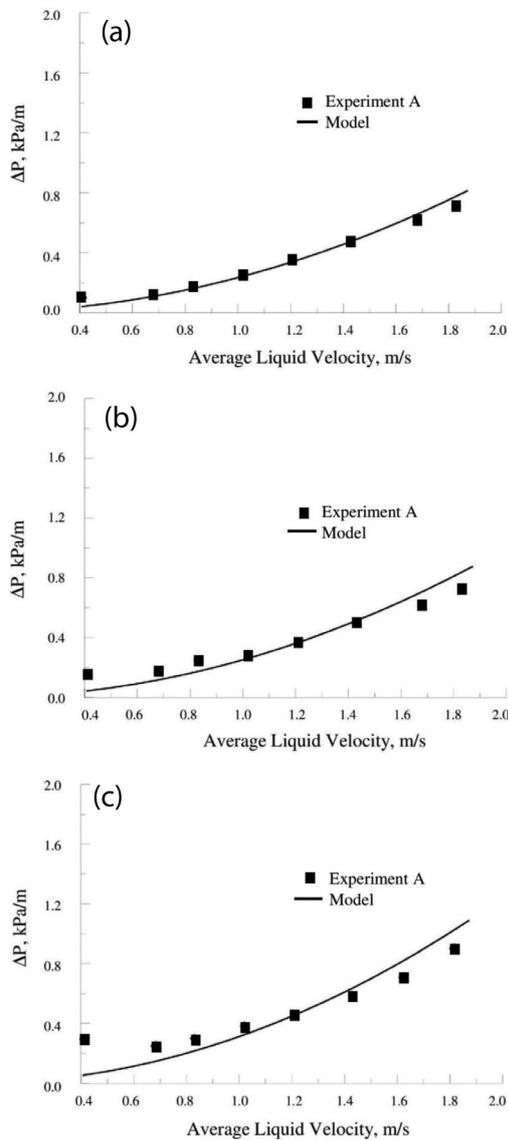


Figure 3: Comparison of simulation results for experimental data set A at different concentrations: a) $C = 0.20$, b) $C = 0.30$, c) $C = 0.50$

from the figure, for the capsule flow of an anomalous shape and having a higher density than the carrier-liquid density, the increasing average velocity of the carrier resulted in an increase in the pressure drop. The simulation results are in good agreement with the experimental results at capsule concentrations of (0.20, 0.30 and 0.50).

The simulation results of pressure-drop calculations give a sufficient accuracy at low capsule concentrations. The anomalous-capsule concentration is greater than 30 %, the solution diverges and the accuracy reduces. Interactions within the phases change the hydrodynamic behavior of the flow. For the pressure-drop simulations at higher concentrations, a separate flow model can be applied. In this study, a homogenous-flow approximation is used in order to have a vision about the variation in the pressure drop of the capsule flow.

The comparison of the simulation results for experimental data set B are given in **Figure 4**. In this set, spherical capsules having a lower density than the water were carried by the flowing fluid in the pipe. As it can be seen from the figure, the simulation results are in good agreement with the experimental data. The increase in the carrier velocity resulted in a higher pressure drop at capsule concentrations of (0.05, 0.10, 0.15 and 0.20). The results of the simulation showed a small discrepancy at capsule concentrations of (0.05 and 0.20) values.

In the two-phase flow of ice-water capsule system, parameters such as suspension-to-wall friction coefficients and solid-phase concentrations ($\phi/\%$) have a significant impact on the analysis of the pressure drop. Different suspension-viscosity models and suspension-to-wall friction coefficients were applied in order to investigate the change in the pressure drop of the two-phase ice-water mixture. The cases analyzed (Case 1 and Case 2) are shown in **Table 2**. In the model validation, Case 2b was considered as the base case. Spherical particles of ice with a 0.08 m diameter and water as the carrier were used in the analyses. The temperature of the carrier was taken as 20 °C for various Cases related to

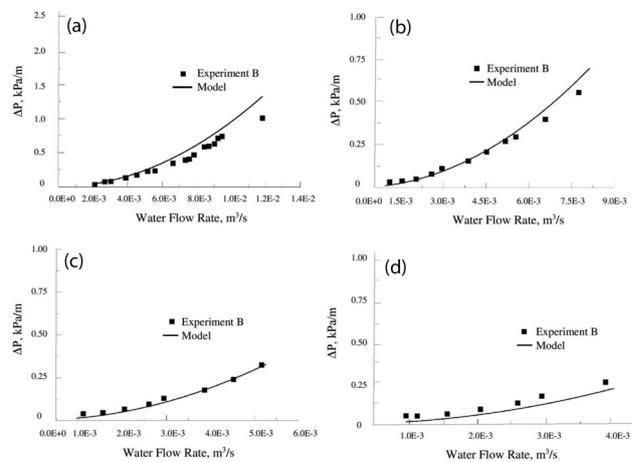


Figure 4: Comparison of simulation results for experimental data set B at different concentrations: a) $C = 0.05$, b) $C = 0.10$, c) $C = 0.15$, d) $C = 0.20$

the suspension viscosity (Cases 1b and 2b) and to the suspension-wall friction factors (Cases 1a and 1b) at capsule concentrations within the range of 0.05–0.25.

Figure 5 presents the impact of the suspension-to-wall friction factor on the pressure drop. As can be seen from the figure, the pressure-drop variation, with respect to the Reynolds number, follows a similar trend in both Cases (1b and 2b). When the Blasius friction-factor correlation was used in the pressure-drop calculations, a deviation from the actual values was observed, especially at high Reynolds-number values.

Table 2: Cases considered in the model^{17,23,24–27}

		Case 1	
		Case 1a	Case 1b
μ_m		$\mu_{Liquid} (1 + 2.5C)$	$\mu_{Liquid} e^{\frac{5(1-\epsilon)}{3\epsilon}}$
f		$\frac{64}{Re_m}$, $Re_m < 2300$	$\frac{0.0791}{Re_m^{0.25}}$, $Re_m \geq 2300$
		Case 2	
		Case 2a	Case 2b
μ_m		$\mu_{Liquid} (1 + 2.5C)$	$\mu_{Liquid} e^{\frac{5(1-\epsilon)}{3\epsilon}}$
f		$2 \cdot \frac{6565}{Re_m^{1.5}}$, $2800 < Re_m < 15000$	$2 \cdot \frac{0.0395}{Re_m^{0.25}}$, $15000 < Re_m < 32000$

The pressure-drop variation, with respect to the empirical suspension-viscosity formulations, is presented in Figure 6 for Cases 1a and 1b. Both formulations showed to have a similar impact on the pressure drop of the ice-water two-phase mixture under consideration. The maximum variation in the pressure drop amounted to 10.7 %. This implies that for the ice-water mixtures both formulations can be applied to homogeneous model calculations. The impact of the suspension viscosity and suspension-to-wall friction on the pressure drop was found to be greater at high values of the Reynolds number whereas no significant impact of these factors on the pressure drop was observed at low values of the Reynolds number.

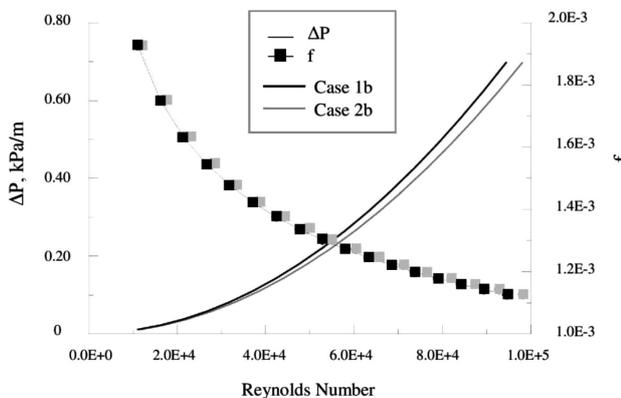


Figure 5: Effect of suspension-wall friction coefficients on the pressure gradient (spherical capsules, $d/D = 0.8$, $C = 0.10$)

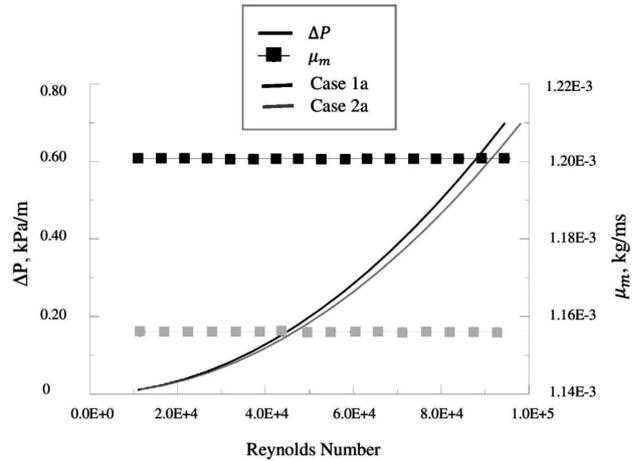


Figure 6: Effect of suspension viscosities on the pressure gradient (spherical capsules, $d/D = 0.8$, $C = 0.10$)

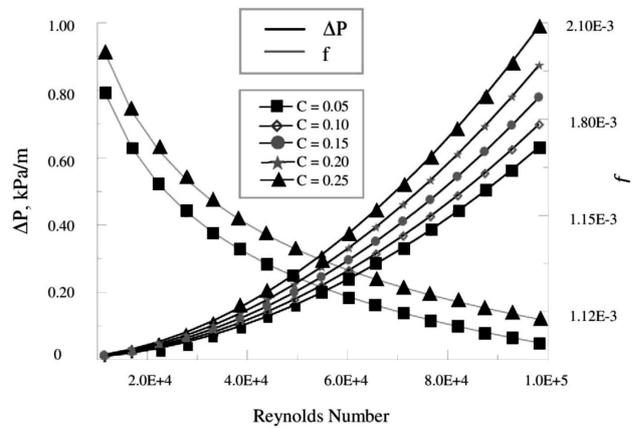


Figure 7: Effect of volumetric solid concentration on the pressure gradient (base case, spherical capsules, $d/D = 0.8$)

The pressure-drop variation with the capsule concentration of the ice-water two-phase mixture is shown in Figure 7. As can be seen from the figure, higher capsule concentrations resulted in higher pressure gradients. The pressure drop also reached higher values with the increase in the Reynolds number. The increase in the concentration raised the impact of the friction factor.

4 CONCLUSIONS

The modelling of two-phase flows is a challenging and complex process, especially for liquid-solid flows. Therefore, a simplified model has to describe the ice-water two-phase flow characteristics with a satisfactory accuracy. In this regard, in the present study, a one-dimensional model was developed as the first approximation. In the simulation model, the pressure drop during the ice-water two-phase-mixture flow inside the pipe is described. The impact of different suspension-viscosity models, suspension-wall friction coefficients and volumetric capsule concentrations on the pressure drop of the capsule flow were investigated throughout

the study. The simulation results were also validated with the results obtained from two different experimental studies. One of the experiments was performed with capsules having an anomalous shape and density greater than the density of the carrier liquid and the other one was performed with spherical-shaped capsules with a density lower than the carrier liquid. In both experiments, measurements were carried out in a horizontal, straight plexiglass pipe and water was used as the carrier. This study revealed that a steady-state, homogenous model approach can be used with confidence for the calculations of the pressure drop of the solid-liquid two-phase mixture in a horizontal pipe. The model results are reliable, especially for the spherical capsules at a volumetric concentration of up to 25 % and for anomalous capsules at concentrations lower than 50 %. The suspension-wall friction factor¹⁷ and suspension-viscosity models²⁷ used in the calculations were found to be applicable to the pressure-drop calculations for solid-liquid two-phase mixtures.

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Nomenclature

<i>A</i>	Cross-sectional area of the pipe (m ²)
<i>f</i>	Suspension-wall friction coefficient (dimensionless)
<i>C</i>	Volumetric concentration of solid phase (dimensionless)
<i>d</i>	Capsule outer diameter (m)
<i>D</i>	Pipe inner diameter (m)
<i>l</i>	Capsule length for anomalous capsule shapes (m)
<i>L</i>	Pipe length (m)
<i>ṁ</i>	Mass flow rate (kg/s)
<i>P</i>	Pressure (Pa)
<i>Re</i>	Modified suspension Reynolds number (m)
<i>U_m</i>	Mean velocity of two-phase mixture (m/s)
<i>θ</i>	Volumetric flow rate (m ³ /s)

Greek symbols

ρ	Density (kg/m ³)
ε	Radially averaged voidage in the pipe(m)
μ	Viscosity (kg/ms)
τ	Shear stress (N/m)

Subscripts

acc	Acceleration
fric	Friction
ice	Ice
m	Mean
w	Water